Passive control of nonlinear single-degree-of-freedom structures utilizing tuned mass damper-inerter

Abdollah Javidialesaadi
Nicholas E. Wierschem
Mark D. Denavit
Department of Civil and Environmental Engineering, University of Tennessee Knoxville, USA

ABSTRACT: This paper investigates the effectiveness of the tuned mass damper-inerter (TMDI) for the vibration control of single-degree-of-freedom structures considering nonlinearities in the structure. The TMDI consists of a tuned mass damper (TMD) with an inerter that is connected between the TMD secondary mass and a fixed support. The TMDI, which has been recently proposed and investigated, has been found to be highly effective, in comparison to a TMD with an equal amount of physical mass, in reducing the dynamic response of structures under various excitations. However, all previous investigations have been conducted with the assumption of a linear base structure. As the real behavior of structures under extreme loads can feature important nonlinearities, the control of the nonlinear response of structures with the TMDI should be considered. In order to study the control of structures featuring nonlinearities with TMDI, the "inerter element" is developed by the authors in the Open System for Earthquake Engineering Simulation (OpenSees) software framework. The results of this work demonstrate that the nonlinearity in the base structure is an important factor affecting the performance of the TMD and TMDI and that the TMDI provides additional robustness when considering these nonlinearities.

1 INTRODUCTION

Traditional tuned mass dampers (TMDs), which consist of a small mass connected to the primary structure through a spring and a dashpot, have been proposed and investigated for passive control of structures (Den Hartog 1985; Warburton 1982). It has been shown that the effectiveness of TMDs is enhanced by increasing the secondary mass to the primary mass ratio (Sadek et al. 1997). Recently, to increase the effectiveness of the TMD without utilizing large physical secondary mass, inerter-based tuned mass dampers have been recently proposed and investigated (Marian and Giaralis 2014; A Javidialesaadi and Wierschem 2018; Garrido, Curadelli, and Ambrosini 2013; Abdollah Javidialesaadi and Wierschem 2018; Hu and Chen 2015). The inerter is a simple mechanical device, such as the ball-screw or rack and pinon, that can produce a large effective mass while utilizing a small physical mass via the transformation of translational motion to the rotational motion of a flywheel (Smith 2002).

The tuned mass damper-inerter (TMDI) is one of the inerter-based tuned mass dampers that has been proposed recently and has broadly demonstrated effectiveness in the reduction of the dynamic response of structures, in comparison to the TMD (Marian and Giaralis 2014). The TMDI consists of a traditional TMD with an inerter that is located between the secondary mass and the ground. The research related to TMDIs has recently been expanded to investigate their usage for utilizing MDOF structures and base isolation systems (Giaralis and Taflanidis 2018; De Domenico and Ricciardi 2017) as well as nonlinear devices similar to the TMDI (Abdollah Javidialesaadi and Wierschem 2019). However, there has been no work similar to what has been done regarding nonlinear structures with TMDs (Zhang and Balendra 2013; Sgobba and Marano 2010), and the primary structure has been considered linear elastic in all previous works related to the TMDI.

This paper investigates the response of a nonlinear SDOF structure equipped with a TMDI. For this purpose, OpenSees (McKenna, Scott, and Fenves 2010) is used for the numerical analysis and an inerter element is developed in this program. To consider the nonlinear behavior of the SDOF structure, its stiffness is modeled using a hardening material. Band-limited white noise ground motion is considered as the excitation and the maximum value of the primary structure's displacement is calculated to evaluate the response of the structure. Comparisons of the performance of the system with a TMDI are made to the response of the system with a TMD. The effects of both the level of inertance and the relative magnitude of the material model's post-yield stiffness on the response of the structure are evaluated.

2 MODEL AND FORMULATION

Figure 1 depicts the nonlinear SDOF structure with a TMDI, subjected to a ground acceleration. In this figure $\ddot{x}_g$ represents the ground acceleration. m_1, $k_{Nonlinear}$, and c_1 represent the mass, nonlinear stiffness, and damping of the primary structure and m_2, b, and c_2 denote the secondary mass, inertance, and the device damping, respectively.

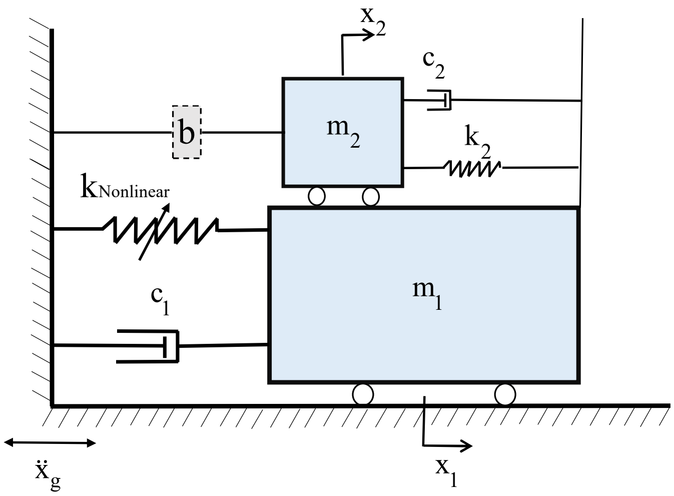

Figure 1: Nonlinear SDOF base structure with TMDI

From the definition of the inerter, the governing equation is given by:

$$F = b(\ddot{x}_j - \ddot{x}_i) \tag{1}$$

Where b is inertance, F is the force across the device, and $\ddot{x}_i$ and $\ddot{x}_j$ are the nodal accelerations of the devices two terminals. It should be noted that the value of the inertance is a function of the flywheel properties and the properties of the mechanism used to produce the rotation; thus, it is possible to produce large values of inertance with a small physical mass. As there is no built-in ability for OpenSees to model inertance, this functionality was added to via a software customization where inertance was optionally added as part of the restoring force of a 2-dimensinal truss element.

Utilizing Eq. (1) and considering the stiffness component of the restoring force of the primary structure as a function of displacement, $f(x_1)$, the equations of motion for the system can be written as:

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + f(x_1) + k_2(x_1 - x_2) = -m_1\ddot{x}_g \\ (m_2 + b)\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = -m_2\ddot{x}_g \end{cases} \tag{2}$$

The term $m_2 + b$ shows that one of the effects of the inertance in this configuration is an increase in the effective secondary mass.

As a simplified analogy for a structure that would yield under extreme loads, the stiffness component of the restoring force of the primary structure is modeled as linear elastic until yield with kinematic hardening after that point (see Figure 2). The "hardening model" in OpenSees is used to produce this behavior and the parameters of the model are a modulus of elasticity = 1 N/m^2, yield force = 10 N, and kinematic hardening modulus = 0.1 N/m^2.

Additionally, a linear case is also examined considering the same modulus of elasticity.

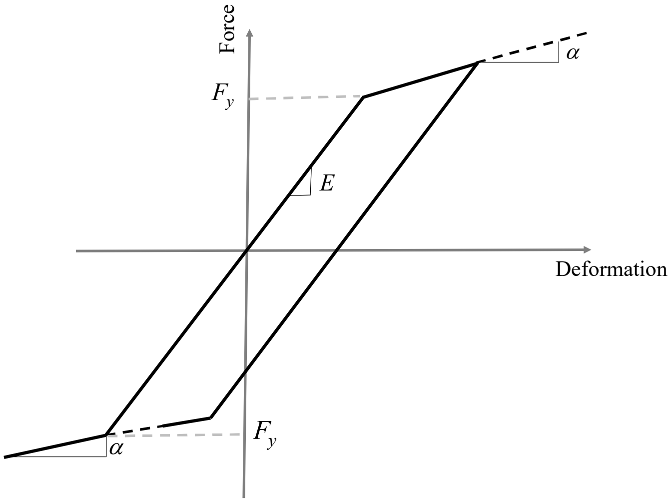

Figure 2: Kinematic hardening model

3 ANALYSIS AND RESULTS

For the analysis in this paper, the primary structure is considered undamped with a frequency of 0.16 Hz (1 rad/s). The mass of the primary structure is equal to one and the secondary mass is considered as 2% of the primary structure mass. Two inertance values, 0.02 and 0.1, for the TMDI are considered.

Band-limited white noise ground acceleration with a cutoff frequency of 5 Hz is considered for the loading of the structure. The duration of 200 seconds is used for the analysis. To average out the effects of the individual ground acceleration time-histories, 500 runs of each analysis are performed, each with a different seed used to create the band-limited white noise loading, and the mean values of the response are utilized for evaluating the performance. Figure 3 shows a sample of the ground acceleration used as an input.

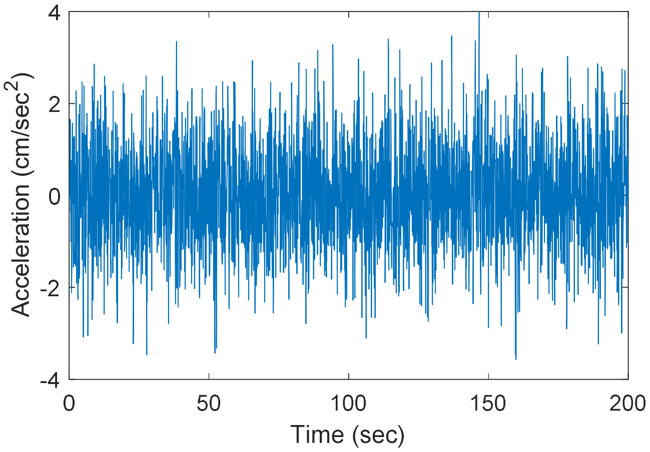

Figure 3: Sample of ground acceleration

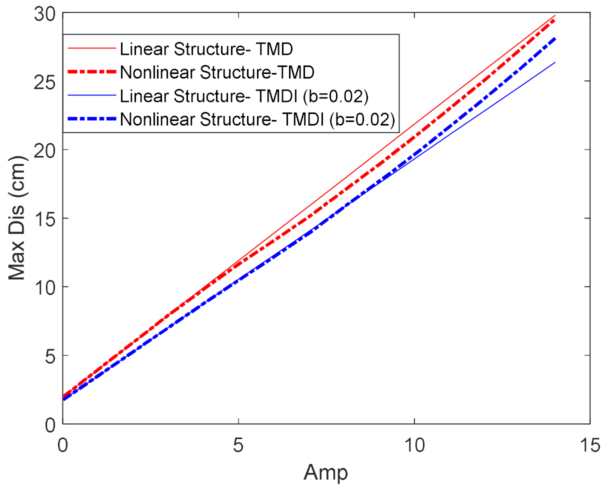

Figure 4: Max displacement of the primary structure vs amplitude scale of the input.

To examine the effects of increases in the amplitude of the loading, these ground motions are linearly scaled upwards. The maximum absolute displacement of the primary structure in 4 cases at each amplitude scale is considered: linear structure with TMD, linear structure with TMDI, nonlinear structure with TMD, and nonlinear structure with TMDI. It should be noted that the optimum design values of the TMD and TMDI are based on the H-infinity optimization of the systems with the TMD or TMDI, respectively. The TMD and TMDI parameter values are held constant at all ground acceleration amplitudes regardless of if the structure is responding linearly or nonlinearly.

Figure 4 shows the variation of the maximum displacement of the primary structure versus the amplitude scaling of the white noise. This figure shows that the response of the linear and nonlinear structures with both the TMD and TMDI are in good agreement when the amplitude is low and the structure is in the linear region. By increasing the amplitude, both the nonlinear structure with the TMD and TMDI show a lower response, compared to their respective linear counterparts. This lower response is due to the contribution of hysteretic damping from the yielded stiffness element to the, otherwise undamped, primary structure. At every amplitude scaling level examined, the response of the system with the TMDI is lower than the response of the system with the TMD

To examine the effect of the inerter on the response of the nonlinear structure with a TMDI, the reduction coefficient R is defined as the ratio of the maximum response of the structure with a TMDI to the maximum response of the structure with a TMD. Figure 5 shows the effect of the inerter on the coefficient R as well.

Figure 5 illustrates that, in all cases the TMDI outperform the TMD. Additionally, the performance of the TMDI significantly increases in the large inertance case examined. In the linear structure, by increasing the amplitude the effectiveness of the inerter does not change. However, in the nonlinear region, by increasing the amplitude of the loading, the inerter's performance advance starts to reduce, but never goes away completely and the TMDI still has superior performance (R<1). This phenomenon can be explained by the effective mass which the inerter adds to the secondary mass of the device. When the effective mass increases, the sensitivity of the structure due to the change in the stiffness and detuning decreases.

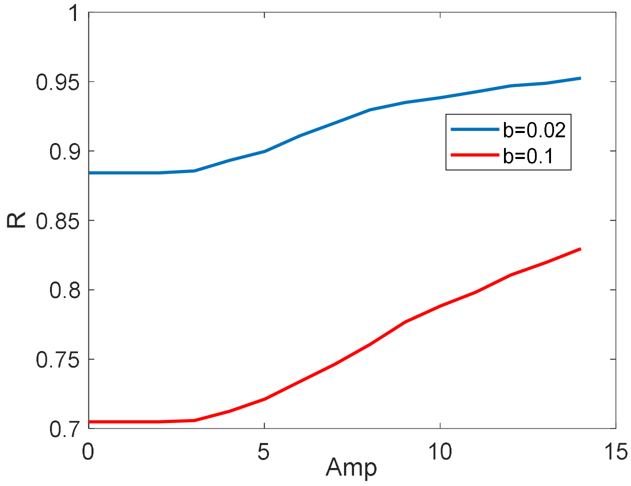

Figure 5: Effect of inerter in the reduction of maximum response amplitude vs ground acceleration amplitude scaling factor

Example time history responses of the nonlinear structure with the TMD and the TMDI with b=0.1 are also presented in Figure 6. As seen in this figure, both cases feature nonlinear responses and the TMDI provides a reduction in overall and peak response, compared to the TMD.

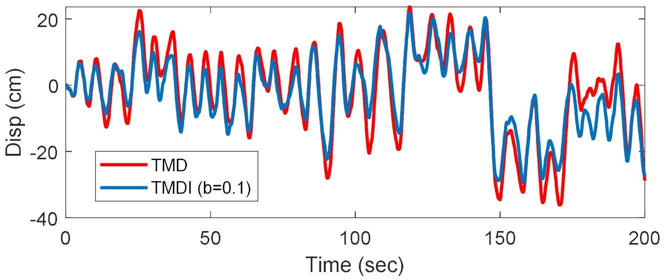

Figure 6: Time history response of structures with TMD and TMDI

4 CONCLUSIONS

This work investigates the response of a single-degree-of-freedom (SDOF) structure considering nonlinearities equipped with a tuned mass damper-inerter (TMDI) and subjected to a band-limited white noise. The effectiveness of the TMDI in comparison to the tuned mass damper (TMD) in the reduction of the response of linear primary structures has been investigated before; however, the response of a nonlinear structure with a TMDI has not been investigated previously. To perform the analysis for this study, an element was modified in the finite element program OpenSees to include inertance. Then a SDOF structure equipped with TMDI is modeled in OpenSees and subjected to band limited white noise. To consider yielding of the structure, the stiffness element of this structure is modeled using an element with kinematic hardening.

The displacement of the primary structure is considered as the output for the evaluation.

The results of this study show that, when the structure was in the linear range, the addition of an inerter makes the TMDI more effective at controlling the response of the structure, compared to the TMD with the same physical mass. As expected, while the structure is in the linear range, the amplitude of the loading does not influence the TMDI's effectiveness. However, when the amplitude of the loading is such that the structure moves into the nonlinear range of its response, the effectiveness of both the TMD and the TMDI is reduced. Even though the effectiveness of the TMDI is reduced, its performance is still superior to the TMD while in this nonlinear response regime. This result can be explained by the increased effective mass of the mass damper due to the inertance, which reduces the sensitivity of the system to the detuning that occurs because of changes in the stiffness of the primary structure in the nonlinear region of its response. As even large values of inertance can be produced with very small amounts of physical mass, the superior performance provided by the TMDI can plausibly be provided at low cost and provide a means for utilizing mass dampers to delay damage to the structure and control the nonlinear response once yielding occurs. The OpenSees element that can represent the effects of inertance and was developed in this work will allow for the investigation of the use of inerter-based control devices in a wide range of realistic structures with nonlinear behavior.

5 REFERENCES

De Domenico, Dario, and Giuseppe Ricciardi. 2017. 'An Enhanced Base Isolation System Equipped with Optimal Tuned Mass Damper Inerter (TMDI): An Enhanced Base-Isolation System Equipped with Optimal Tuned Mass Damper Inerter (TMDI)'. *Earthquake Engineering & Structural Dynamics*, December. https://doi.org/10.1002/eqe.3011.

Den Hartog, JP. 1985. *Mechanical Vibrations*. New York: Dover Press.

Garrido, Hernán, Oscar Curadelli, and Daniel Ambrosini. 2013. 'Improvement of Tuned Mass Damper by Using Rotational Inertia through Tuned Viscous Mass Damper'. *Engineering Structures* 56 (November): 2149–53. https://doi.org/10.1016/j.engstruct.2013.08.044.

Giaralis, A., and A.A. Taflanidis. 2018. 'Optimal Tuned Mass-Damper-Inerter (TMDI) Design for Seismically Excited MDOF Structures with Model Uncertainties Based on Reliability Criteria'. *Structural Control and Health Monitoring* 25 (2): e2082. https://doi.org/10.1002/stc.2082.

Hu, Yinlong, and Michael Z.Q. Chen. 2015. 'Performance Evaluation for Inerter-Based Dynamic Vibration Absorbers'. *International Journal of Mechanical Sciences* 99 (August): 297–307. https://doi.org/10.1016/j.ijmecsci.2015.06.003.

Javidialesaadi, A, and N. E. Wierschem. 2018. 'Three-Element Vibration Absorber–Inerter for Passive Control of Single-Degree-of-Freedom Structures'. *Journal of Vibration and Acoustics* 140 (6): 11. https://doi.org/10.1115/1.4040045.

Javidialesaadi, Abdollah, and Nicholas E. Wierschem. 2018. 'Optimal Design of Rotational Inertial Double Tuned Mass Dampers under Random Excitation'. *Engineering Structures* 165 (June): 412–21. https://doi.org/10.1016/j.engstruct.2018.03.033.

Javidialesaadi, Abdollah, and Nicholas E. Wierschem. 2019. 'An Inerter-Enhanced Nonlinear Energy Sink'. *Mechanical Systems and Signal Processing* 129 (August): 449–54. https://doi.org/10.1016/j.ymssp.2019.04.047.

Marian, Laurentiu, and Agathoklis Giaralis. 2014. 'Optimal Design of a Novel Tuned Mass-Damper–Inerter (TMDI) Passive Vibration Control Configuration for Stochastically Support-Excited Structural Systems'. *Probabilistic Engineering Mechanics* 38 (October): 156–64. https://doi.org/10.1016/j.probengmech.2014.03.007.

McKenna, Frank, Michael H. Scott, and Gregory L. Fenves. 2010. 'Nonlinear Finite-Element Analysis Software Architecture Using Object Composition'. *Journal of Computing in Civil Engineering* 24 (1): 95–107. https://doi.org/10.1061/(ASCE)CP.1943-5487.0000002.

Sadek, Fahim, Bijan Mohraz, Andrew W. Taylor, and Riley M. Chung. 1997. 'A Method of Estimating the Parameters of Tuned Mass Dampers for Seismic Applications'. *Earthquake Engineering and Structural Dynamics* 26 (6): 617–636.

Sgobba, Sara, and Giuseppe Carlo Marano. 2010. 'Optimum Design of Linear Tuned Mass Dampers for Structures with Nonlinear Behaviour'. *Mechanical Systems and Signal Processing* 24 (6): 1739–55. https://doi.org/10.1016/j.ymssp.2010.01.009.

Smith, M.C. 2002. 'Synthesis of Mechanical Networks: The Inerter'. *IEEE Transactions on Automatic Control* 47 (10): 1648–62. https://doi.org/10.1109/TAC.2002.803532.

Warburton, G. B. 1982. 'Optimum Absorber Parameters for Various Combinations of Response and Excitation Parameters'. *Earthquake Engineering & Structural Dynamics* 10 (3): 381–401.

Zhang, Z., and T. Balendra. 2013. 'Passive Control of Bilinear Hysteretic Structures by Tuned Mass Damper for Narrow Band Seismic Motions'. *Engineering Structures* 54 (September): 103–11. https://doi.org/10.1016/j.engstruct.2013.03.044.